(1)

Partiel mars 2012

1/ 1) a) effet Zeeman (1)

b) Validité cadre perturbatif $\hat{H}_0 \gg V_B^{(1)}$ (1)

$\vec{M} = -\gamma \vec{L}$ (1) avec $\gamma = \frac{e}{2m_e}$

3/ $V_B^{(1)} = -\vec{M}.\vec{B} = \gamma \vec{L}.\vec{B} = \gamma B \hat{L}_3 = \omega_c \hat{L}_3$ (1) ; $\hat{L}_3 \Rightarrow$ valeur propre $m_\ell \hbar$

c) n nbre quantique principal, ℓ nbre quantique azimuthal, m_ℓ nbre quantique magnétique (1)

$n \in \mathbb{N}^*$, $0 \leq \ell \leq n-1$; $-\ell \leq m_\ell \leq +\ell$ (1)

3/ $n = 2 \Rightarrow \ell = 0$ (état 2s) $\Rightarrow m_\ell = 0$

$\Rightarrow \ell = 1$ (états 2p) $\Rightarrow m_\ell = 0 \pm 1$ (1)

3/ d) diagonaux car $|n \ell m_\ell\rangle$ vecteur propre de $\hat{L}_3$ (1) $\Rightarrow$ qd projection sur bra : termes non nuls

$$E^{(1)}_{2\ell m_\ell} \Rightarrow E^{(1)}_{200} = 0 \quad \text{car } m_\ell = 0$$

$$\Rightarrow \begin{cases} E^{(1)}_{210} = 0 & \text{car } m_\ell = 0 \\ E^{(1)}_{211} = \hbar\omega_c & m_\ell = +1 \\ E^{(1)}_{21-1} = -\hbar\omega_c & m_\ell = -1 \end{cases}$$ (2)

4/ 2) a) effet Stark (1) $\vec{p} = -e\vec{r}$ (1)

$V_B^{(1)} = -\vec{p}.\vec{E} = e\vec{r}.\vec{E} = exE$ (1)

avec $x = r \sin\theta \cos\varphi$ (1)

6/ b) $E^{(1)}_{2\ell' m_\ell', 2\ell m_\ell} = eE \underbrace{\int_0^\infty R_{2\ell'}(r)\, r^3 R_{2\ell}(r)\, dr}_{\langle R_{2\ell'}(r)|r|R_{2\ell}(r)\rangle = -\frac{6\pi\varepsilon_0\hbar^2}{m_e e^2} n\sqrt{n^2-\ell^2}} \iint Y^*_{\ell' m_\ell'}(\theta,\varphi) \sin^2\theta \cos\varphi\, Y_{\ell m_\ell}(\theta,\varphi)\, d\theta\, d\varphi$ (2)

qd $\ell' = \ell \pm 1$, $m_\ell' = m_\ell \pm 1$

$\Rightarrow$ 4 termes non nuls (1)

$\langle 211 | V_B^{(1)} | 200 \rangle$

$\langle 200 | V_B^{(1)} | 211 \rangle$

$\langle 21-1 | V_B^{(1)} | 200 \rangle$

$\langle 200 | V_B^{(1)} | 21-1 \rangle$

②

calcul du 1er terme $\langle 2\,1\,1 | V_B^{(1)} | 2\,0\,0 \rangle$

③ partie angulaire $\int_0^{\pi}\int_0^{2\pi} Y_1^{1*}(\theta,\varphi) \sin^2\theta \cos\varphi \, Y_0^0(\theta,\varphi) \, d\theta \, d\varphi$

$= \int_0^{\pi}\int_0^{2\pi} \left(-\sqrt{\frac{3}{8\pi}} \sin\theta \, e^{-i\varphi}\right) \sin^2\theta \cos\varphi \left(\frac{1}{\sqrt{4\pi}}\right) d\theta \, d\varphi$

$= \frac{-\sqrt{3}}{\pi\sqrt{32}} \int_0^{\pi} \sin^3\theta \, d\theta \int_0^{2\pi} \cos\varphi \, e^{-i\varphi} \, d\varphi = \frac{-\sqrt{3}}{\pi\sqrt{32}} \times \left(\frac{4}{3}\right) \times (\pi) = \frac{-1}{\sqrt{3}\sqrt{2}}$

partie radiale $\langle R_{21}(r) | r | R_{20}(r) \rangle = -\frac{6\pi \varepsilon_0 \hbar^2}{m_e e^2} \times 2\sqrt{2^2 - 1^2} = -\frac{12\sqrt{3}\pi \varepsilon_0 \hbar^2}{m_e e^2}$

d'où $E^{(1)}_{211\,200} = \cancel{e} E \times \left(-\frac{12\cancel{\sqrt{3}}\pi \varepsilon_0 \hbar^2}{m_e e^{\cancel{2}}}\right) \times \left(-\frac{1}{\cancel{\sqrt{3}}\sqrt{2}}\right) = \frac{12\pi}{\sqrt{2}} \frac{\varepsilon_0 \hbar E}{m_e e} \hbar$

$\times \alpha$

3) $V_T^{(1)} = V_B^{(1)} + V_E^{(1)}$

$$= \hbar \begin{array}{c} \begin{array}{cccc} |211\rangle & |210\rangle & |21-1\rangle & |200\rangle \end{array} \\ \begin{pmatrix} \omega_L & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_L & -\alpha \\ \alpha & 0 & -\alpha & 0 \end{pmatrix} \end{array}$$

②

5

$\Rightarrow$ eq caractéristique

$d^2(\omega_L^2 + 2\alpha^2 - d^2) = 0$

$\Rightarrow d^2 = 0 \Rightarrow$ 2 énergies nulles doublement dégénérées ①

$d^2 = \pm\sqrt{\omega_L^2 + 2\alpha^2} \Rightarrow E_\pm = \pm\hbar\sqrt{\omega^2 + 2\alpha^2}$

②